Statistical mechanics

Lecture note: 4

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Topics Covered:

1. Bose Einstein statistics
2. Comparison of different distribution functions:
3. Bose Einstein Condensation:
4. Problems

UNIT: Bose Einstein statistics

A. Bose Einstein Distribution Formula

1. Bosons are indistinguishable integral spin particles such as photons, 4He atom, phonons, magnons, π-mesons.
2. Bosons have symmetric wave function
3. The occupation no. for the i^{th} energy level is given as

$$f = \frac{1}{e^{\frac{\varepsilon_i - \mu}{kT}} - 1}$$

4. 'n' is called occupation number.
5. Bose gas: follows Bose Einstein statistics.

B. Comparison of different distribution functions:

$$\frac{n_i}{g_i} = f = \frac{1}{e^{\frac{\varepsilon_i - \mu}{kT}} + \kappa}$$

Where

statistics	κ	
MB	0	Classical
BE	-1	Quantum
FD	1	Quantum

At sufficient higher energies classical and quantum results are identical.

If ε_i is very large, $e^{\frac{\varepsilon_i-\mu}{kT}}$ >>> 1 and +1 and -1 in the denominator is neglected, The BE stat and FD stat converges to MB stat.

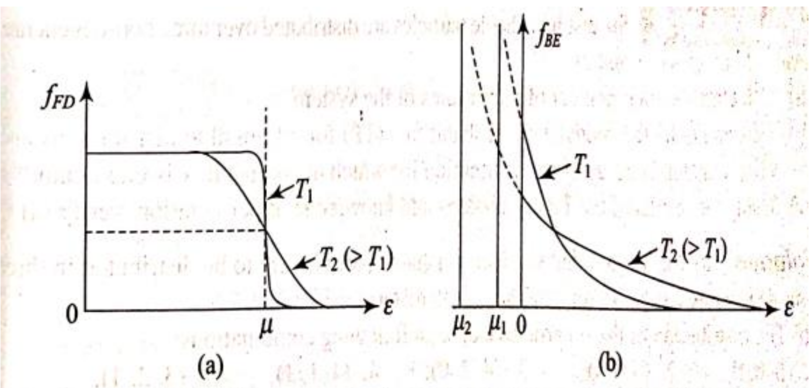
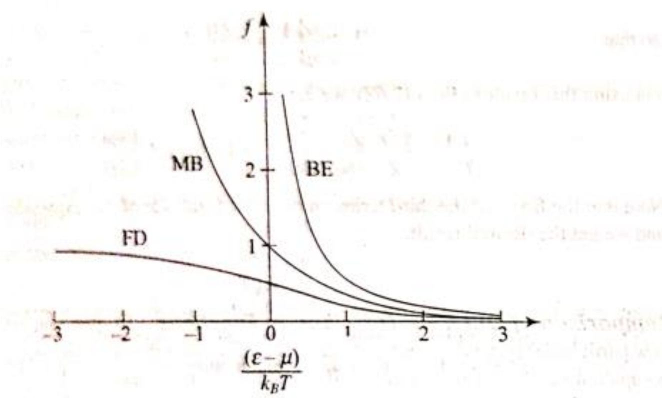


Fig. 12.13 (a) Plot of f_{FD} versus ε for fermions at two temperatures T_1 and T_2 (> T_1) and (b) Plots of f_{BE} versus ε at two different temperatures for bosons.



Plots of distribution function for M-B, F-D and B-E statistics versus $(\varepsilon_i - \mu)/k_B T$ for fixed T and μ.

C. Bose Einstein Condensation:

When the temperature of a Bose-Einstein gas is lowered below the critical temperature Tc, the no. of particles in the ground state rapidly increases. This rapid increase in the population of the ground state below the critical temperature Tc for Bose-Einstein gas is called Bose Einstein Condensation.

$$f = \frac{1}{e^{\frac{\varepsilon_i - \mu}{kT}} - 1} \qquad (1)$$

For continuous distribution

$$n(\varepsilon) = \frac{1}{e^{\frac{\varepsilon_i - \mu}{kT}} - 1} \qquad (2)$$

No. of particles within the element phase space $d^3r\, d^3p$ is product of no. of quantum states and the occupation number.

No. of quantum states for energy ε = G* $d^3r\, d^3p/h^3$, where G is intrinsic angular momentum.

No. of quantum states: $$g(\varepsilon)d\varepsilon = \frac{G*2\pi V*(2m)^{\frac{3}{2}}*\varepsilon^{1/2}d\varepsilon}{h^3} \qquad (3)$$

Thus no. of particles in phase space $d^3r\, d^3p$ is

$$dN = \frac{G*2\pi V*(2m)^{\frac{3}{2}}*\varepsilon^{\frac{1}{2}}*d\varepsilon}{h^3} \frac{1}{e^{\frac{\varepsilon_i - \mu}{kT}} - 1}$$

$$dN = CV \frac{\varepsilon^{\frac{1}{2}}*d\varepsilon}{e^{\frac{\varepsilon_i - \mu}{kT}} - 1}$$

Where C = $\frac{G*2\pi*(2m)^{\frac{3}{2}}}{h^3}$

Thus, total no. of particles $$N = CV \int_0^\infty \frac{d\varepsilon}{e^{\frac{\varepsilon_i - \mu}{kT}} - 1}$$

$$N = CV \int_0^\infty \frac{d\varepsilon}{\frac{1}{A} e^{\frac{\varepsilon_i}{kT}} - 1} \qquad (4)$$

<u>Task1</u>: to find the no. of quantum states for ε=0

From equation 3, no. of quantum states α $\varepsilon^{1/2}$, so for ε=0, no of quantum state is zero. While actually when ε=0, no. of quantum state is 1. Thus equation 4 does not take into account, total no. of particles present at ground level i.e. ε=0. So to calculate total no of particles we will take ε=0 case separately.

Let N_g denotes no. of particles at ground level than equ. 4 can be written as,

$$N = N_g + CV\int_0^{\infty}\frac{d\varepsilon}{\frac{1}{A}e^{\frac{\varepsilon_i}{kT}}-1}$$

$$N = N_g + N_{ex}$$

Where $N_g = \frac{1}{\frac{1}{A}-1} = \frac{d\varepsilon}{e^{\frac{-\mu}{kT}}-1}$

And $N_{ex} = CV\int_0^{\infty}\frac{d\varepsilon}{\frac{1}{A}e^{\frac{\varepsilon_i}{kT}}-1}$

Task2: To calculate value of N_{ex}:

Let ε/KT = x, then dε= KTdx

$$N_{ex} = CV(kT)^{3/2}\int_0^{\infty}\frac{x^{1/2}dx}{\frac{1}{A}e^{x}-1}, \quad (5)$$

For completely degenerate Boson gas A=1 (μ=0).

And $\int_0^{\infty}\frac{x^{1/2}dx}{\frac{1}{A}e^{x}-1} = \xi(3/2)\Gamma(3/2) = 2.612*(\pi^{1/2}/2)$ (6)

Combining the results in eqn. (5), we get

$$N_{ex} = 2.612V\left(\frac{2\pi mkT}{h^2}\right)^{3/2} \quad (7)$$

Thus equation 7 is a case of maximum degeneracy (A=1). This equation gives maximum no. of particles, at any temperature the excited energy levels can hold. Thus, at any temperature T, if total no. of particles are greater than N_{ex}, the excess particles pushed back to ground state and this is called Bose Einstein Condensation.

References:

1. *Thermal Physics by S.C. Garg, R.M. Bansal and C. K. Ghosh.*
2. *Statistical Physics, Berkley series volume 5.*
3. *Fundaments of Statitical Mechanics and Thermal Physics, F. Rief.*

Tutorial questions:

1. Find out the total no. of ways of filling 3 particles in two energy groups of 4 cells each, so that 2 particles are placed in one energy level and one particle in the other energy level such that, particles are

   (i) Identical and distinguishable.

   (ii) Identical and indistinguishable.

   (iii) Identical and indistinguishable and follow Pauli Exclusion Principle.

2. Find out the condensation temperature for He II given that the volume occupied by 1 g mole of the gas is 27.4 cm^3.

3. Seven bosons are arranged in two compartments. The first compartment has 8 cells and the second compartment has 9 cells of equal size. What is the total no. of microstate for a macrostate when 3 bosons are in one compartment and 4 bosons are in other compartment?

4. Bose-Einstein condensation occurs in a weakly interacting particle at temperature 2.17 K at a certain density. If density of the system is 1000 times smaller, at what temperature will the transition takes place? In which of the two isotopes of potassium, Potassium 40 (which has 19 protons and 21 neutrons) and potassium 41 (which has 19 protons and 22 neutrons) will such condensation takes place? Give reasons.